From Mrs. Walton's Algebra I class

The Quadratic Quarterly

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Inside this issue:

Did you know?

- Quadratic Equations have two solutions that may be ***Real*** or *Complex*
- The Discriminant for a quadratic equation tells us whether it has two, one or no ***Real*** solutions.

"Extreme Makeover" Relies on Woodbridge Students

Once again the Woodbridge High School Algebra class has come to the rescue of the designers from Television's hottest program, "Extreme Makeover-Home Edition". This time, the design team came to the class begging them to solve their dilemma: They had a room that was 9m x 12m and wanted to put a rug in the room that would leave an equal amount of wood flooring around the edges of the room and would also have an area equal to 1/2 the entire room. Time was limited and the Woodbridge team accepted the challenge.

The first step, for this fine group of students, was to diagram the problem. It quickly became clear that the length and width of the rug would need to be:

Length: 12—2x   Width: 9—2x

Since the area of the whole room was (9 * 12) = 108 m^2 the astute students quickly realized that the equation to be built from this situation was a quadratic:

$$(12-2x) * (9-2x) = 1/2\ (108)$$

which, when put into standard form, became:

$$2x^2 - 21x + 27 = 0.$$

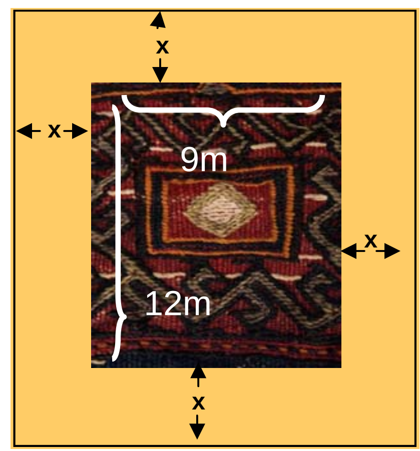

Diagram of Room

Recognizing that there were several ways to solve this quadratic equation to find the amount of space to be left on all sides of the rug, the students chose the Quadratic equation method (See related article on page 2) and discovered that the value for x that made this equation true was x = 1.5 m. The rug would need to be 10.5 m x 7.5 m.

The design team was ecstatic and gave the class front row tickets to the final day of filming. These talented students have a bright future in Hollywood and beyond!

Quadratic Formula is Not New...

Given how pivotal the Quadratic Formula was to solving the Extreme Makeover—Home Edition's critical problem, knowledgeable citizens need to know that this formula is not new. In fact, the solutions to quadratic equations have been of interest to many cultures over the millennia. There is evidence that ancient Babylonians were solving quadratic equations around 2000 B.C. in ways quite similar to the one students used above. Over the years following this, notable contributions were made by ancient Chinese and Egyptian scholars. Early Arab mathematician Mohammed ibn Musa al-Khowarizmi wrote a book published in 825 A.D. that showed the solving of equations in a manner close to the Quadratic Formula. Other scholars over the centuries, such as Egyptian Abu Kamil, Persian Omar Khayyam, French Descartes and German Gauss built upon the early foundations so that by the late-1700's mathematics had a body of knowledge known as Classical Algebra.

Mathematical markings on ancient Babylonian tablet

Squares Awares ^{2}

Woodbridge High School
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From Mrs. Walton's Algebra I class

All you ever needed to know about Quadratics…. And more!

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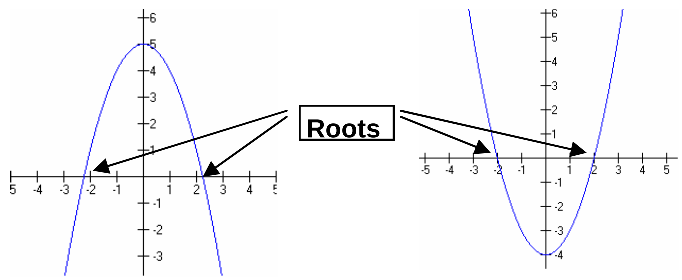
Quadratics Primer

In each issue of the Quadratic Quarterly we choose to explain one mathematical concept in very simple terms. In this issue we will define the basics of a Quadratic equation and the significance of its graph.

A Quadratic Equation is an equation that uses just one variable (ex., "x") and has one term that has an "x^2" in it. There can be no larger exponent than "x^2" in a quadratic equation. An example of a quadratic equation in standard form is: $2x^2 - 21x + 27 = 0$. (This is the equation that the Woodbridge HS students used for determining the area around a rug on page 1.)

Every quadratic equation graphs into the shape of a ***parabola,*** something like a "U". Here are two examples:



Y = -x^2 + 5 **Y = x^2 - 4**

The places where the graphs cross the x axis are called ***"roots" or "solutions".*** It is these values of x that make the equation true. If the coefficient of the x term is positive, the parabola opens upwards and if the coefficient of the x^2 term is negative, the parabola opens downwards.

Quadratic Formula - Unmasked

As complex as the Quadratic Formula appears, it is derived simply

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

from the standard representation of a quadratic equation using the technique of "completing the square".

Dividing our quadratic equation by a, we have:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

which is equivalent to:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

Now we can "complete the square". To do this we need to add a constant (something that does not depend on x) to the expression on the left side of the "=", that will make it a perfect square trinomial in the form:

$$x^2 + 2xy + y^2$$

Since "$2xy$" in this case is $(b/a)x$, we set them equal to each other and solve for y and we discover that $y = b/(2a)$. We can then square this value to get "y^2" and add it to both sides which gives us:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}.$$

The left side is now a perfect square. It is the square of

$$(x + b/(2a)).$$

The right side can be written as a single fraction; the common denominator is $4a^2$. We get:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

We can take the square root of both sides of this equation and get:

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}.$$

Subtracting $b/(2a)$ from both sides, we get:

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$